# Aeroacoustics of Supersonic Jet Flows from a Contoured Plug-Nozzle

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**The results of an experimental study of the acoustic far-field, the shock-associated noise, and the nature of the repetitive shock structure of supersonic jet flows issuing from a plug-nozzle having an externally expanded contoured plug with a pointed termination, operated at a range of supercritical pressure ratios $\xi \doteq 2.0$ to 4.5, are reported. The supersonic jet flow from the contoured plug-nozzle (CPN) is shown to be shock free and virtually wakeless at pressure ratio $\xi \doteq 3.60$ (fully expanded jet flow Mach number $M_j \doteq 1.49$). As compared with the noise characteristics of underexpanded jet flows from an "equivalent" convergent nozzle, substantial reductions in the overall sound pressure levels (OASPL's) are achieved at all observation angles in the entire range of the pressure ratios. The typical bucket-type behavior of the OASPL vs $M_j$, characteristic of supersonic jet flows from the contoured convergent-divergent (CD) nozzle, is not observed for the CPN operated over a range of supercritical pressure ratios. At the off-design conditions, the noise reductions for the CPN are higher than for the equivalent contoured CD nozzle. Moreover, the noise intensity of the CPN nozzle jet flows scales to the second power of the shock-strength related parameter $\beta = (M_j^2 - 1)^{1/2}$ and not to the fourth power as observed for the underexpanded jet flows from a convergent round nozzle. Similarly, the OASPL's of the CPN scale to $\beta_*^2 = M_j^2 - M_d^2$ and not to $\beta_*^4$ as reported for the contoured CD nozzle where $M_d$ is the design Mach number of either the CPN or the contoured CD nozzle.**

## Nomenclature

$A$ = area
$K$ = annulus radius ratio ($R_P/R_N$)
$M$ = Mach number
$p$ = pressure
$R$ = radius (also, radial distance from the nozzle exit to the measuring location)
$R_N$ = radius of the convergent nozzle lip
$R_P$ = radius of the plug at the sonic point
$\alpha$ = inclination of the convergent nozzle lip to the nozzle axis
$\beta$ = parameter, $\sqrt{(M_j^2 - 1)}$
$\beta_*$ = $\sqrt{|M_j^2 - M_d^2|}$
$\nu$ = Prandtl-Meyer angle
$\psi$ = inclination of the surface of the contoured plug at the sonic point
$\xi$ = ratio of reservoir absolute pressure to the ambient pressure
$\theta$ = azimuthal angle of the location of measurement (angle between the line joining the center of the nozzle exit to the center of the microphone and the downstream jet flow axis)

*Subscripts*

$d$ = design condition
$e$ = exit condition
$j$ = fully expanded jet flow
$t$ = throat (sonic condition)

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## Introduction

THE repetitive shock structure is inherently present in supersonic jet flows from improperly expanded nozzles. The passage of flow fluctuations through such repetitive shock structure results in the generation of shock-associated broadband noise[1-3] and feedback-induced screech noise.[4,5] At high supercritical pressure ratios, the noise-generating mechanisms of the major sources of the radiated noise from supersonic jet flows (e.g., the mixing and the shock-associated noise components) are often coupled.[6] Normally, the intensity of the shock-generated acoustic radiation is directly dependent upon the shock strength and the level and the coherence of the flow fluctuations convected through the shock front.[1,4,7-9] Therefore, to suppress aerodynamic noise components radiated by improperly expanded single-stream supersonic jet flows, the extent, the spacing, and the strength of the repetitive shock structure and the level and coherence of the jet-flow fluctuations convected through the shock fronts need to be modified, such that the overall strength of the noise-contributing sources and the effectiveness of their noise-generating mechanisms are reduced. The use of a contoured convergent-divergent (CD) nozzle often is considered favorably as a design option for attenuating the shock-associated noise component generated by the exhaust flow of a modern high specific thrust jet engine. The shock-associated noise component is eliminated if the exhaust flow of a contoured CD nozzle operated at its design pressure ratio is shock free. However, in practice, such exhaust nozzles of necessity are operated over an extended range of pressure ratios where at the off-design pressure ratios in either the over- or the underexpanded mode, the repetitive shock structure is formed in the exhaust flows. At low supercritical pressure ratios, the shock structure may even be present in the diverging part of the contoured CD nozzle. The overall sound pressure levels as a function of the fully expanded jet-flow Mach number $M_j$ of supersonic jet flows issuing from the over- and the underexpanded CD nozzles have been shown to be significantly lower than those of an equivalent (i.e., of the same mass flow rate, pressure ratio, and the exhaust area) single round convergent nozzle.[2]

Some recent acoustic studies of supersonic jet flows from plug-nozzles have also shown appreciable noise-suppression effects.[10-13] In some of these studies, the plugs were rather long cylindrical center bodies,[10,11] and in others the plug surface was either uncontoured (conical) and/or truncated.[13] The repetitive cellular shock structure is necessarily formed in the supersonic jet flows from such plug nozzles. Moreover, such comparatively long plugs are likely to have the related aerodynamic and weight penalties. Therefore, to circumvent some of these disadvantages and to eliminate or to weaken the repetitive shock structure in supersonic jet flows, the use of a short contoured externally expanded plug with a pointed termination suggests itself as an attractive alternative. The aeroacoustics of such short contoured plug-nozzles have not been studied before. In the present studies, the results of the far-field noise characteristics and the corresponding observations of the gasdynamics of the jet flows of such a short contoured plug-nozzle (CPN) having a pointed termination operated in the over-, the fully, and the underexpanded modes of operation over a range of supercritical pressure ratios appropriate for practical jet engine applications are compared with those from both an equivalent contoured CD nozzle and an equivalent convergent nozzle. In the shock-free and virtually wakeless supersonic jet flows emanating from a CPN operated at its design pressure ratio, the noise-generating mechanism is primarily due to turbulent mixing of the free jet flow. The far-field acoustic data of shockless supersonic jet flows of such CPN flows also serve as the baseline acoustic spectral data for a comparative assessment of 1) the shock-associated noise when the CPN is operated at off-design pressure ratios, and 2) the noise-suppression effectiveness of the improperly expanded supersonic jet flow issuing from equivalent conical plug-nozzles of other geometries, configurations, and terminations.

## Prediction of the Isentropic Plug Contour

A plug-nozzle is a modification of a conventional CD nozzle. It combines a convergent nozzle and an externally expanded contoured plug where the supersonic expansion downstream of the sonic throat of the convergent nozzle occurs externally over the plug surface; unlike the CD nozzle, the flow in part is controlled by the ambient back pressure and not by the nozzle walls. Therefore, downstream of the CPN throat the free boundary of the jet flow is self-adjusting. At the design pressure ratio, the CPN jet flow at the plug-nozzle exit is uniform, axial, and shockless (see Fig. 1).

The design of the plug contour for an externally expanded plug-nozzle is based on the following key considerations.

1) The expansion waves are assumed to be centered at the lip of the convergent nozzle of the CPN (Fig. 1). For the free jet-flow boundary at the lip to be straight and parallel to the nozzle axis,the convergent wall has to have an inclination $\alpha$, which is equal and opposite to the Prandtl-Meyer angle $\nu$ for the design Mach number $M_d$.

2) The individual expansion waves emanating from the nozzle lip and incident on the plug surface are all canceled by suitable local compression turns provided at the contoured plug surface. The last expansion wave of the *P-M* expansion (corresponding to the design Mach number $M_d$) must end at the plug tip (or apex) and is straight, being the start of the uniform simple flow region. The plug contour as such is a streamline of the potential (isentropic) flow issuing from the plug nozzle, thus resulting in a plug of the shortest possible length. A methodology of designing an isentropic supersonic inlet-plug using the method of characteristics (MOC) was developed by Connors and Meyer.[14] To avoid the computational difficulty near the flow centerline (the radius $R \to 0$), they assumed a finite plug-tip angle at the inlet and a finite-strength oblique shock extending from the inlet-plug tip to the nozzle lip at its throat. The plug contours were predicted for relatively high design pressure ratios (or high design Mach numbers $M_d$). It was suggested by them that this approach for the inlet-plug design also applies to the prediction of the plug contour of an externally expanded plug nozzle. For a convergent nozzle of a given exit radius $R_N$, the maximum length of the contoured plug $L_{max}$ is fixed for a given design Mach number $M_d$. Moreover, for an ideal contoured plug-nozzle, the annulus-radius ratio $K$ (ratio of the plug radius $R_p$ at the sonic point to the nozzle lip radius $R_N$) is a unique function of the design Mach number.[15,16] Thus, at high design pressure ratios (or high design Mach numbers), the corresponding values of $K$ are also large, resulting in small annulus width $W_t$ of the throat of the isentropic plug-nozzle. Therefore, the assumption made by Connors and Meyer[14] that the sonic line at the throat is straight is reasonably satisfied at high design pressure ratios. The present aeroacoustic studies, however, are aimed at plug-nozzles to be operated at lower design pressure ratios normally encountered in turbojet engines for supersonic jet propulsion. This range of lower pressure ratios was not covered in the earlier predictions of the contours of isentropic inlet-plugs by Connors and Meyer. At lower design pressure ratios $\xi_d$, the annulus-radius ratio $K$ of the plug-nozzle are smaller, and consequently the annulus width $W_t$ of the throat of the plug-nozzle would be comparatively larger. Because of the considerably different slopes of the inner and outer walls at the plug-nozzle throat, the flow at the throat is converging and essentially nonuniform. This, therefore, would result in an appreciable curvature of the sonic line. Consequently, instead of Connors and Meyer method that prescribed a straight sonic line at the throat, a prediction of the exact curved sonic line is necessary for obtaining a truly isentropic flow of a minimum length contoured plug-nozzle.

The design parameters that must be determined for the start of the solution are the annulus-radius ratio $K$, the inner wall (i.e., the plug) slope $\psi$ at the sonic point, and the shape of the sonic line. Consider the flow near the throat in an infinitesimal annulus of an axisymmetric plug-nozzle (see insert in Fig. 1).

Its annular throat area is given by

$$A_t = \pi(R_1^2 - R_2^2)/\cos\left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$\omega_1 \doteq \omega_2$$

The geometrical condition of Mach number gradient at the throat in the stream direction being zero,

$$\frac{dA_t}{dL} = 0 \quad \text{gives} \quad \omega_2 = \sin^{-1}\left|\frac{\sin\omega_1}{k}\right|$$

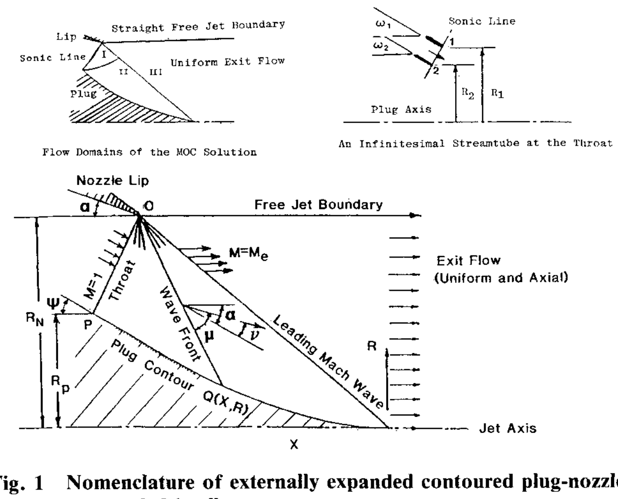



**Fig. 1 Nomenclature of externally expanded contoured plug-nozzle and fully expanded jet flow.**

where the variable radius ratio of the infinitesimal annular element $k = R_2/R_1$. The mean flow direction of the streamline in the annular element is

$$\omega_m = \frac{\omega_1 + \omega_2}{2}$$

$$k = \left| 1 - \frac{\cos\omega_m}{(A_e/A_t)} \right|^{1/2}$$

where $(A_e/A_t)$ is given by the area Mach number relation for an isentropic flow.

The preceding set of relations may be used for establishing an iterative scheme starting from the nozzle lip, to obtain the shape of the curved sonic line along with the values of the geometrical configuration parameter $K$ of the plug and the slope of the plug at the sonic line $\psi$. The starting value of $\omega_1$ is given by

$$\omega_1 - \alpha = \nu(M_d)$$

where $\nu$ is the Prandtl-Meyer angle, and $M_d$ is the design Mach number. The iterative scheme is initiated by guessing a value of $k$, and the process is so rapidly convergent that the points on the curved sonic line are easily obtained.

Having obtained the sonic line shape, the annulus-radius ratio $K$ and the initial slope of the plug at the sonic line $\psi$, the method-of-characteristics (MOC) was used to obtain the plug profile, which is a bounding stream surface of the plug-nozzle potential flow. The method consisted of treating the flow in three domains (see Fig. 1) where the domain I is a mixed region having both left-running and right-running characteristics. The domain II is a simple region having only one family of characteristics. The lines of constant characteristics, starting from the nozzle lip, were traced in 15 small intervals, and the plug profile was generated in successive stages by providing suitable local compression turnings at the surface to cancel the incident expansion waves. (For an alternate (approximate) method for a nearly isentropic plug contour developed in the course of the present investigation, see Refs. 15 and 16.) It may be noted that a line of constant characteristic is necessarily curved in an axisymmetric flow. A combination of computational and graphical approach was used to predict the isentropic plug contour. For the coordinates of the contoured plug, see Ref. 15.

## Plenum-Chamber and Contoured Plug-Nozzle

The design Mach number $M_d$ of the CPN is 1.5. The resulting annulus-radius ratio $K$ and the plug-surface angle $\psi$ at the sonic point are 0.43 and 28.4 deg, respectively. An extremely sharp, pointed termination (thickness of only 0.25 mm) of the contoured plug tip was provided to reduce the wake flow and thus practically eliminate the possibility of recompression shock that normally will originate from any base flow downsream of the plug termination.

The details of the stainless steel plenum chamber for the plug-nozzle jet rig are shown in Fig. 2. The plenum chamber has an overall length of 72.06 cm, and the ratio of the convergent nozzle inlet to the exit area is 46. The flow velocity in the cylindrical part of the plenum chamber when the convergent nozzle of exit diameter $D = 4.5$ cm is choked is noted to be about 7 m/s. This insured that the intensity level of the noise generated by the flow in the plenum chamber is comparatively negligible. The relevant details of the overall geometry of the externally expanded contoured part of the plug and the preceding hump are also shown in Fig. 2. The plug was mounted axially aligned with a slender cylindrical plug holder. Mechanism was provided to insure accurate axial alignment of the plug holder in three transverse directions. A wire screen of 50% porosity is installed downstream of the plug holder to smooth the flow by breaking up any large upstream eddies that may be present in the compressed air supply system and/or in the plenum chamber. The inner profile of the stainless steel convergent round nozzle is finely

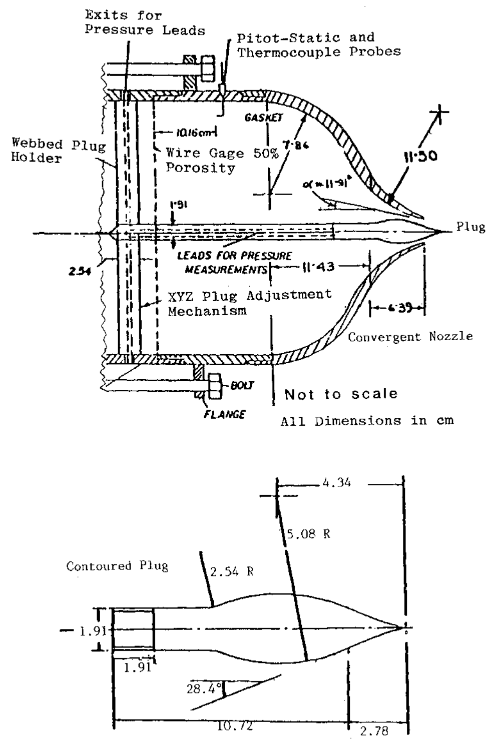
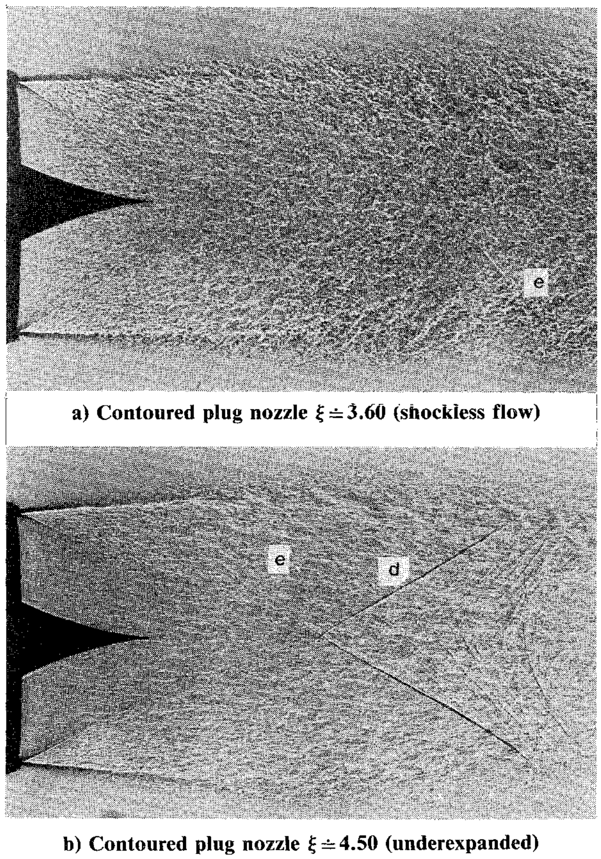



Fig. 2 Plenum chamber, plug mounting assembly, and plug geometry.




a) Contoured plug nozzle $\xi \doteq 3.60$ (shockless flow)

b) Contoured plug nozzle $\xi \doteq 4.50$ (underexpanded)

Fig. 3 Typical spark shadowgraphs of jet flows from the contoured plug nozzle: d—shock formed from the reflections of expansions unintercepted by the plug; e—weak compression wave.

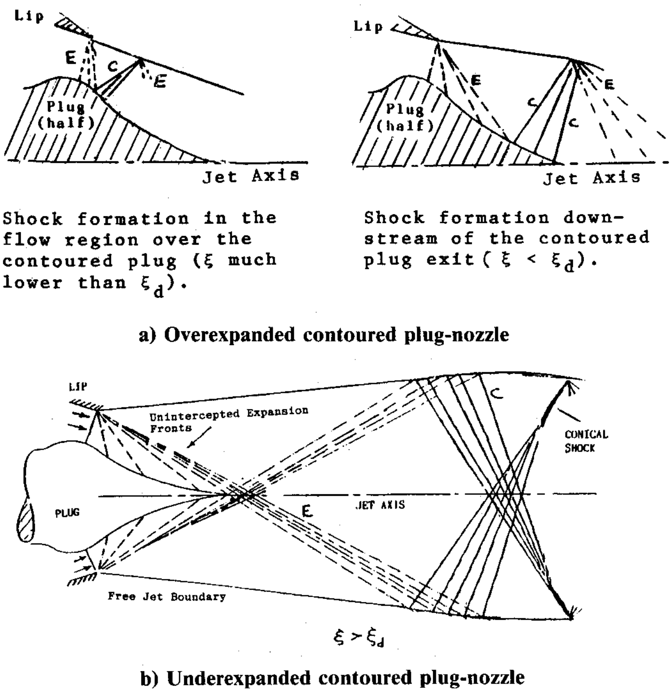



**Fig. 4 Sketch of the wave structure of supersonic jet flows from contoured plug nozzle: E—expansion waves – – –; C—compression waves____.**

polished. To avoid acoustic reflections and screech generation, a fairly sharp convergent nozzle lip of thickness 0.5 mm was provided. The compressed air supply system and the nozzle up to its lip was covered with fiberglass. For additional details of the actual installation of the convergent nozzle and the stainless steel plenum chamber for the jet rig of the plug nozzle, see Refs. 15 and 16.

## Acquisition and Extent of Experimental Data

The acoustic spectral data were recorded in an anechoic chamber with free-space dimensions, from wedge tip to wedge tip, of approximately 8 m×6.5 m×4.3 m. The chamber is anechoic down to a frequency of 150 Hz. One-third octave sound pressure level spectra of the far-field noise radiated by supersonic jet flows from the CPN and the convergent nozzle were recorded at eight equally spaced locations, 15 deg apart between azimuthal angles $\theta = 15$–$120$ deg, measured with reference to the downstream jet-flow axis on an arc of radius $R = 3.05$ m. For a convergent nozzle of exit diameter $D \simeq 4.5$ cm, the microphone location is at $R/D \approx 65$. The microphone positioning is accurate within $\pm 1$ deg. Standard B and K acoustic instrumentaion was used to record the one-third octave sound pressure level spectral data over the frequency range $f = 200$–$100$ kHz. The acoustic specral and the spark shadowgraphic data of the supersonic jet flows issuing from the contoured plug-nozzle and from the basic convergent nozzle (i.e., the plug-nozzle operated without the plug and its mounting assembly) were recorded at a range of pressure ratios $\xi = 2.0$–$4.5$ (or the range of the fully expanded Mach number $M_j$ of 1.05–1.67).

For analysis, the lossless one-third octave spectral data were obtained by applying the microphone and the atmospheric absorption (humidity) corrections[17] to the recorded one-third octave sound pressure level spectra. The quality of the acoustic spectral data was validated by showing that the acoustic intensity of subsonic jet flow from the basic convergent round nozzle has nearly $U^8$ dependence.[16] The overall sound pressure levels (OASPL's) were calculated at each measuring location by summing the corrected one-third octave band sound pressure levels (SPL's) over the frequency range 200–50 kHz.[15] To underscore the physical features of the supersonic jet flows of the contoured plug-nozzle and their shock structure and to facilitate the interpretation of the experimental observations of the radiated noise, spark shadowgraphs of the supersonic jet flows from the CPN were recorded. Typical spark shadowgraphs are shown in Figs. 3a and 3b.

## Pertinent Features of Supersonic Jet Flows of the Contoured Plug-Nozzle

The spark shadowgraphs of the CPN jet flows at different pressure ratios show that at pressure ratio $\xi \doteq 3.6$ (or $M_j \doteq 1.49$) the supersonic jet flow in the plug region and further downstream is free of any repetitive shock structure (Fig. 3a). Moreover, the free jet-flow boundary just downstream of the nozzle lip is straight and parallel to the axis of the plug-nozzle, further indicating that the supersonic jet flow is fully expanded. Also for $M_e \doteq 1.49$ (shockless flow condition), the area ratio $A_e/A_t$ as measured from the shadowgraph of the fully expanded jet flow agrees well with that calculated from the area Mach number relation for an isentropic supersonic flow. Because of the very pointed plug termination, there is hardly any wake flow evident in the spark shadowgraph of the fully expanded plug-nozzle flow. Also, the growth of the boundary layer over the smooth polished contoured plug surface is not evident in the shadowgraphic record. Therefore, it may be assumed to be extremely thin. The absence of any recompression shocks from the wake flow insures that downstream of the pointed plug apex, the base flow effects are negligible. In such fully expanded plug-nozzle flows, the sources and the mechanisms for the generation of the shock-associated noise component are considered to be absent, and only the mixing noise is radiated. Therefore, the noise radiated by such shockless and virtually wakeless plug-nozzle supersonic jet flow provides ideal baseline acoustic spectral data for comparative assessment of the reduction of the shock-related noise component of improperly expanded jet flows issuing from equivalent plug nozzles of different contours, configurations, and modes of operation.

The operation of the CPN at less than the design pressure ratio (i.e., $\xi < \xi_d$ or $M_j < M_d$) is designated here as the overexpanded mode of operation. For details of the flow features of jet flows from the overexpanded contoured plug-nozzle and their explanation, see Ref. 15. All of the expansion waves between the leading and the tail wave fronts of the centered expansion fan emanating from the nozzle lip are canceled at the contoured plug surface. However, the compression-turning of the contoured plug surface downstream of the location where the tail Mach fronts of the expansion fan are incident on the plug generates a family of compression wave fronts (Fig. 4a). These may coalesce and form a conical oblique shock in the plug region of the flow. Either the oblique shock or the compression fronts reflect as expansions from the free jet boundary, which in turn reflect as compressions from the opposite jet boundary. These reflected compressions coalesce and may lead to the formation of an oblique shock in the plug nozzle jet flow. To meet the constant pressure condition at the free jet-flow boundary, further reflections result in weak repetitive shock cells. Such oblique shock structure formed by the coalescence of the compression wave fronts originating from only a part of the plug surface is weaker than the oblique shock structure of an underexpanded jet flow of a convergent nozzle operated at the same pressure ratio but formed by the coalescence of the reflected compression waves of the entire Prandtl-Meyer expansion fan from the free jet boundary. As the operating supercritical pressure ratio for the overexpanded mode of operation of the CPN is lowered, the compression waves generated by the compressive turn of the contoured plug surface downstream of (and relatively close to) the CPN throat may lead to the formation of oblique shocks in the plug region (see flow sketch in Fig. 4a). For the appearance of the oblique shock in the plug region at lower pressure ratios, see the spark shadowgraphs of the CPN jet flows in Ref. 15.

The spark shadowgraph of the jet flows from an underexpanded CPN operated at $\xi = 4.5$ is shown in Fig. 3b. All of the expansion waves generated in underexpanded CPN jet flows (up to the design pressure ratio) incident on the contoured plug surface are canceled. The part of expansion wave fronts of the centered expansion fan from the lip of the nozzle corresponding to $\xi > \xi_d$ are not intercepted by the plug and reflect as compression fronts from the opposite free jet-flow boundary to form oblique shock structure farther downstream (see shock front d in Fig. 3b and the corresponding flow sketch in Fig. 4b. (For a comparative study, an uncontoured conical short plug of the same surface area as the contoured plug was incorporated in a plug-nozzle of the same annulus-radius ratio $K = 0.43$. The ratio of the plug length $L_{max}$ from the sonic point to the plug tip to the nozzle radius $R_N = 0.97$ instead of 1.30 for the contoured plug. At higher than the design pressure ratios ($\xi > \xi_d$), the conical plug-nozzle flowfield had two families of repetitive cellular shock structures; for details see Ref. 15.) The corresponding underexpanded jet flow from converging round nozzle operated at $\xi = 4.5$ develops a Mach disk with the attendant mixed subsonic and supersonic flow regions downstream of the normal and the oblique shocks, respectively, of the Mach reflection.

## Assessment of Noise-Suppression Effectiveness of Contoured Plug-Nozzle at Supercritical Pressure Ratios

The typical one-third octave SPL spectra for the fully expanded mode of operation of the CPN at pressure ratio $\xi \doteq 3.60$, the overexpanded mode at $\xi = 3.05$, and the underexpanded mode at $\xi = 4.5$ recorded at $\theta = 90$ deg are compared with the corresponding SPL's of the convergent nozzle operated at the same pressure ratio in Fig. 5. The SPL's of the CPN operated at the design and at the off-design pressure ratios are significantly lower than those of the model convergent nozzle in the entire range of the one-third octave band-center frequencies. In the one-third octave SPL spectra recorded at various angular locations for plug-nozzles operated at a range of pressure ratios, there was no evidence of sudden sharp spectral peaks of the type normally associated with the presence of screech tones.

The dominant noise-generating mechanism for the fully expanded (shockless) jet flows (at the design pressure ratio) is due to turbulent mixing, and it is only at the off-design operating pressure ratios that a combination of the shock-associated and the turbulent-mixing noise is present. To assess the effectiveness of a CPN as a noise suppressor for improperly expanded jet flows, acoustic directivity (OASPL vs $\theta$) is compared with that of an equivalent convergent nozzle, each operated at the same pressure ratio. To obtain OASPL's for the model convergent nozzle of exit diameter $D \simeq 4.5$ cm equivalent to those of the CPN, the calculated OASPL's for the model convergent nozzle were scaled down by normalizing with respect to the throat area of the CPN. For the present plug-nozzle configuration with a low annulus-radius ratio $K$, this correction is only 0.68 dB.

The variation of OASPL's vs $\theta$ of the CPN and the equivalent convergent nozzle operated at a range of pressure ratios $\xi \doteq 3.60$ (fully-expanded), $\xi = 3.05$ (overexpanded), and $\xi \doteq 4.5$ (underexpanded) are compared in Figs. 6a-6c, respectively. The OASPL's increase with increasing pressure ratios. For each of the operating pressure ratios, the OASPL's of CPN are nearly constant at observer angles 60 deg $\leq \theta \leq$ 120 deg. The OASPL's at all $\theta$ are substantially lower than those for an equivalent convergent nozzle operated at the same pressure ratio. The reductions in OASPL of CPN are observed both at the lower angles to the jet-flow axis where the turbulent-mixing noise component is dominant as well as at higher angles to the jet-flow axis, where the shock-associated

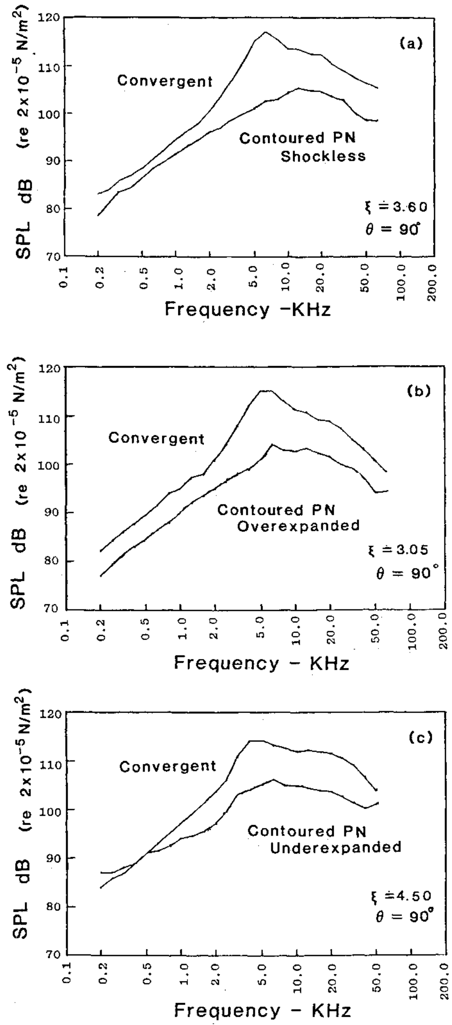



**Fig. 5 Typical one-third octave sound pressure level spectra of jet flows from convergent nozzle and contoured plug-nozzle.**

noise component is dominant. When these directivity data for different pressure ratios are cross plotted as $\Delta$ OASPL vs $\theta$, where $\Delta$ OASPL = OASPL for CPN − OASPL for the equivalent convergent nozzle, the levels of noise suppression vary with $\theta$ and the operating pressure ratio, in the range −6 dB to −14 dB. For the CPN at design pressure ratio, the maximum $\Delta$ OASPL $\doteq -9.0$ dB at $\theta = 15$-30 deg and at $\theta = 90$-120 deg, $\Delta$OASPL $\doteq -8.0$ dB.

The variations of OASPL's at $\theta = 90$ deg for the CPN and an equivalent convergent nozzle operated at a range of pressure ratios $\xi = 2.00$–4.5 ($M_j = 1.05$–1.67) are plotted as a function of the logarithmic shock parameter $\log_{10} \beta$ where $\beta = \sqrt{M_j^2 - 1}$ (Fig. 7). For the equivalent convergent nozzle jet flows, it follows the Harper-Bourne and Fisher $\beta^4$ scaling. For the contoured plug-nozzle operated in the overexpanded range of pressure ratios ($\xi < \xi_d$; $M_j < M_d$), OASPL varies as $\beta^{1.8}$. Over the underexpanded range ($\xi > \xi_d$; $M_j > M_d$), it varies as $\beta^{2.7}$. The OASPL over the entire range of $M_j$ varies approximately as $\beta^2$. Therefore, the OASPL's for the CPN increase comparatively less steeply than those for the convergent nozzle. Underlying physical reason for this result is provided by the earlier observation that, when operated at the same pressure ratio, the strength of the shock structure in the jet flows from the over- and underexpanded modes of operation of the CPN is weaker than that in the underexpanded jet flows from an equivalent convergent nozzle. Consequently, the contributions of the shock-associated noise component for the

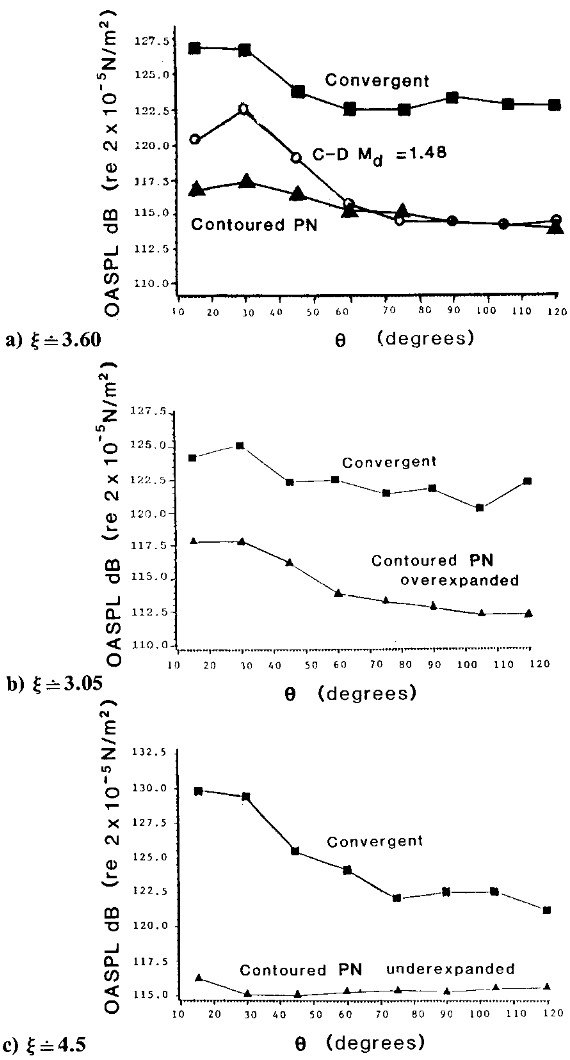



**Fig. 6 Comparison of overall sound pressure level variations with the azimuthal angle of jet flows from the equivalent convergent, equivalent contoured convergent-divergent and contoured plug-nozzles.**

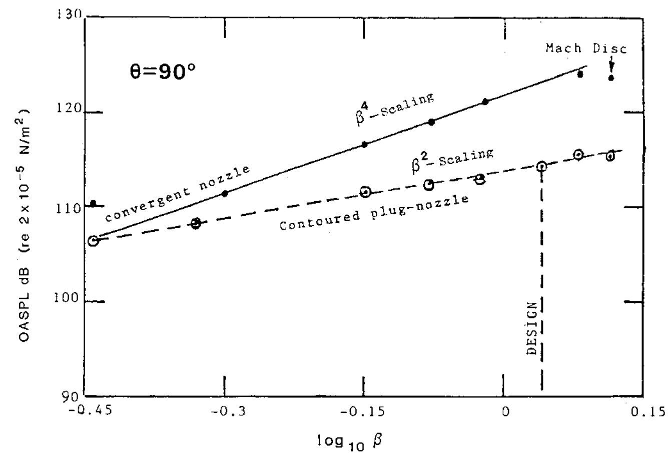



**Fig. 7 Overall sound pressure level variation with the logarithmic shock strength parameter $\beta^2 = M_j^2 - 1$, of supersonic jet flows from the contoured plug-nozzle and the equivalent convergent nozzle.**

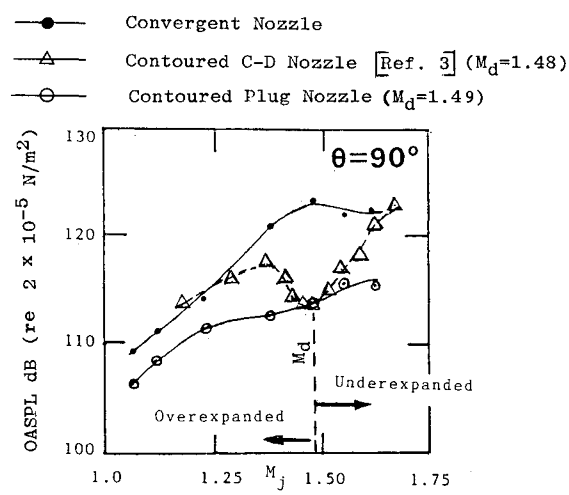



**Fig. 8 Overall sound pressure level as a function of the fully expanded flow Mach number of supersonic jet flows from the contoured plug nozzle, an equivalent contoured convergent-divergent nozzle and an equivalent convergent nozzle.**

CPN jet flows are comparatively weak. The variations of OASPL's at $\theta = 90$ deg and $\theta = 120$ deg at a range of pressure ratios of the contoured *P-N* and the equivalent convergent nozzle are very similar.[18] Therefore, any deductions about the shock-associated noise based on the acoustic data recorded either at $\theta = 90$ deg or $\theta = 120$ deg are equally viable.

## Comparison with an Equivalent Contoured CD Nozzle

The acoustic performance of the CPN ($M_d \doteq 1.49$) was also compared with an equivalent contoured convergent-divergent nozzle of very nearly the same design Mach number ($M_d \doteq 1.48$). Both flows were shock free. The acoustic directivity data (OASPL vs $\theta$) for contoured CD nozzle were taken from an earlier study by Yu and Dosanjh[3] and for equivalence with the CPN the exit area of the CD nozzle was scaled up. The comparison of OASPL's vs $\theta$ of CPN with an equivalent contoured CD nozzle operated at the same design Mach number is shown in Fig. 6a. The OASPL's of the CPN and the contoured CD nozzle are essentially the same for $\theta \geq 60$ deg. However, for $\theta < 60$ deg, the OASPL's for the CPN are noticeably lower than those for the contoured CD nozzle ($\Delta$ OASPL = OASPL of CD nozzle − OASPL of CPN $\doteq 5$ dB at $\theta = 30$ deg). This means that at low $\theta$, the acoustic intensity levels of the mixing noise from the shock-free supersonic jet flows from the CPN are lower than those from an equivalent contoured CD nozzle. The Mach number of the respective uniform shockless supersonic jet flows at the exit of the CPN and the contoured CD nozzle is the same. However, because of the contoured plug as a centerbody, the uniform flow at the exit of the CPN is achieved by externally expanding the free annular jet flow between the throat and the exit, with its attendant gradients in the flow Mach number between the plug surface and the final expansion wave. For the contoured CD nozzle, the expanding of the flow is confined within the contoured nozzle walls. The observed differences in the acoustic directivity at low $\theta$, therefore, may be attributed to the differences in the nature of the annular supersonic turbulent jet flows of the CPN and those of the contoured CD nozzle.

The variations of the OASPL's at $\theta = 90$ deg for the CPN, an equivalent contoured CD nozzle, and an equivalent convergent nozzle are compared for a range of fully expanded flow Mach numbers $M_j$ (Fig. 8). At higher angles to the jet axis, the noise intensity vs $M_j$ for a contoured CD nozzle operated at a range of pressure ratios in the overexpanded mode through the fully expanded to the underexpanded mode, respectively, shows a pronounced "bucket," i.e., the OASPL rapidly falls to a minimum at the design Mach number and equally rapidly rises for higher pressure ratios.[2,3] The appearance of the typical bucket in OASPL vs $M_j$ plot for the OASPL data at $\theta = 90$ deg for the contoured CD nozzle of design Mach number $M_d = 1.48$ studied by Yu and Desanjh[3] is very similar to the one observed by Tanna and Tam[2] for the contoured CD nozzle of $M_d = 1.67$. The absence of such a well-defined bucket in OASPL vs $M_j$ for the CPN, and the striking differences between the OASPL of a CPN and an equivalent CD nozzle of almost the same design Mach number $M_d \doteq 1.48$ as that of the CPN, are demonstrated in Fig. 8. The OASPL of CPN increases with increasing $M_j$ from over-, through the fully, to the underexpanded mode of operation. Around the design Mach number $M_d \doteq 1.49$, only a slight leveling off (at most, a very shallow dip) in the OASPL is seen. However, for the contoured CD nozzle jet flows at $M_j$, which are either noticeably larger than or lower than $M_d$, the OASPL's are observed to approach those of the equivalent convergent nozzle. Moreover, for the CPN jet flows, the intensities of the radiated noise are considerably lower than those of the improperly expanded jet flows from the equivalent contoured CD nozzle and the equivalent convergent nozzle in almost the entire range of supercritical pressure ratios (or $M_j$). It is only at the design Mach number that the OASPL's at $\theta = 90$ deg of the CPN and the equivalent contoured CD nozzle jet flows are very nearly the same (Fig. 6a and Fig. 8).

The OASPL's at $\theta = 90$ deg for the CPN operated over a range of pressure ratios $\xi = 2.0$–$4.5$ (i.e, $M_j = 1.05$–$1.67$) are also plotted against $\beta_*^2 = (M_j^2 - M_d^2)$ in Fig. 9. The parameter $\beta_* = (|M_j^2 - M_d^2|)^{1/2}$ is similar to the one used in Ref. 2 to plot the intensity of the shock-associated noise from a contoured CD nozzle.

The straight line variation of OASPL's as a function of $(M_j^2 - M_d^2)$ shows that except at very low supercritical pressure ratios ($M_j < 1.2$), the OASPL's vary as $\beta_*^2$, which is noticeably less steep than the $\beta_*^4$ variation for the shock-associated noise intensity reported for the contoured CD nozzle jet flows.[2]

The observed differences in the combined intensity of the mixing and shock-associated noise components from a contoured CD nozzle and a CPN as a function of the fully expanded jet-flow Mach number $M_j$ may be explained on the basis of the differences in the physical development of the shock structure in an off-design overexpanded mode of operation of the CPN and the contoured CD nozzle. When the operating critical pressure ratios for the contoured CD nozzle are so low that a normal shock front is present at the nozzle exit, or perhaps even inside the nozzle, the OASPL's are relatively high, approaching those of an underexpanded jet flow from an equivalent convergent nozzle operated at the same pressure ratio. As the operating pressure ratio of the CD nozzle is increased, the exit pressure of the overexpanded nozzle is lower than the ambient pressure, requiring the formation

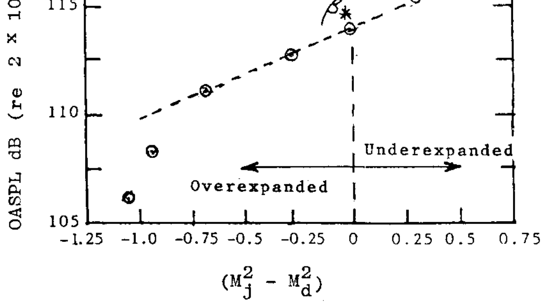



**Fig. 9 Overall sound pressure level scaling as a function of $\beta_*^2 = (M_j^2 - M_d^2)$ of a supersonic jet flow from a contoured plug-nozzle.**

of an oblique shock at the nozzle lip (overexpanded mode) to meet the pressure condition at the free jet-flow boundary; thus, with the increasing pressure ratio, the strength of the oblique shock progressively decreases. The shock structure disappears at the design pressure ratio. This results in the observed minimum in the OASPL at $M_j = M_d$ (Fig. 8). When the pressure ratio is increased beyond the design pressure ratio, the oblique shock structure of a progressively increasing strength reappears in the underexpanded jet flow, and the OASPL increases. Thus, the OASPL vs $M_j$ for a contoured CD nozzle develops the characteristic bucket. To raise the exit flow pressure to the back pressure for the overexpanded mode of the contoured CD nozzle, the oblique shock fronts at the nozzle lip are formed abruptly. However, for the off-design pressure ratio of the overexpanded contoured plug ($M_j < M_d$), the jet flow expands through the centered expansion fan at the lip of the convergent nozzle. In the CPN jet flows, the expansion fan incident on the contoured plug surface is canceled. As discussed earlier (see Fig. 4a), the oblique shocks may be formed by the coalescence of the compression wave fronts, originating from only a part of the plug surface where the local flow Mach number is lower than the design Mach number of the plug. Moreover, the coalescence of the compression fronts into an oblique shock is spatially spread out. This process of oblique shock formation results in comparatively weak repetitive oblique shock structure in the overexpanded CPN jet flows. Therefore, the shock-related component of the OASPL from the overexpanded mode of operation of the contoured plug-nozzle does not vary as steeply with $M_j$ as it does for the jet flows from an overexpanded contoured CD nozzle.

For the underexpanded mode of operation of the CPN, the unintercepted expansion rays from the nozzle lip reflect from the free-jet boundary as compression waves resulting in the formation of the oblique shocks. The mechanism of the formation and the strength of the repetitive shock structure for the CPN, and the corresponding ones for the underexpanded mode of operation of the contoured CD nozzle, are quite similar. Therefore, the appearance of the bucket in the OASPL vs $M_j$ plot, a sharp rise in the OASPL of the CD nozzle in the underexpanded mode ($M_j > M_d$), and the absence of the same for the CPN are difficult to reconcile. However, one may surmise that the presence of the plug as a centerbody and the resulting annular jet flow somehow moderate the potential acoustic sources in the underexpanded turbulent jet flows, such that the increase in the intensity of the radiated noise at $M_j > M_d$ is less steep.

## Conclusions

The noise levels radiated by the fully expanded (shockless) jet flows from an externally expanded contoured plug-nozzle with a pointed plug termination, when operated at its design pressure ratio, are substantially lower than those of underexpanded jet flows from an equivalent convergent nozzle. The noise suppression effects of the CPN operated at design Mach number (shock-free flow) are of comparable magnitude to those of an equivalent contoured CD nozzle operated at the same design Mach number at directions $\theta \geq 60$ deg and are larger at $\theta < 60$ deg. At the off-design pressure ratios, the levels of radiated noise from the improperly expanded jet flows from the CPN are noticeably lower than those from an equivalent contoured CD nozzle operated at the same pressure ratio.

The variation of OASPL as a function of fully expanded jet-flow Mach numbers $M_j$ for the CPN, operated at a range of pressure ratios, does not exhibit a well-defined bucket behavior normally observed for supersonic jet flows from a contoured CD nozzle operated in its overexpanded, through fully expanded, to underexpanded modes of operation. The development of the supercritical jet flows from a convergent round nozzle, an overexpanded contoured CD nozzle, and a CPN results respectively, in repetitive shock structures of different configurations and strengths. Consequently, at higher angles ($\theta = 90$ deg), OASPL of the supersonic jet flows from the CPN operated at a range of pressure ratios follow $\beta^2$ scaling, instead of the Harper-Bourne and Fisher $\beta^4$ scaling for the underexpanded jet flows from a convergent circular nozzle. Similarly, OASPL's of CPN scale approximately as $\beta_*^2$ in comparison with the $\beta_*^4$ scaling reported by Tam and Tanna[2] for supercritical jet flows from the contoured CD nozzle. As such, the plug-nozzle is an attractive design option to reduce the intensity of shock-related noise components radiated from improperly expanded jet flows.

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